An Approximate Formula for Estimating the Peak Value of Lightning-Induced Overvoltage Considering the Stratified Conducting Ground

Qilin Zhang, Member, IEEE, Liang Zhang, Xiao Tang, and Jinge Gao

Abstract—In this paper, we present an improved and extended approximate formula for estimating the peak value of lightning-induced voltages in an overhead line, considering the horizontally stratified conducting ground. The approximate formula proposed in this paper is based on the return stroke transmission-line model (TL) and a trapezoidal lightning return stroke current waveform with the typical representative front time of 3.8 µs and the return stroke velocity of 120 m/µs according to CIGRE and the IEEE Standard 1410 Guide. The extended approximate formula is validated by using the 2-D finite-different time-domain method and Agrawal field-line coupling model. The results show that the proposed approximate formula in this paper is simple and suitable for estimating the lightning-induced voltage peak value considering the horizontally stratified ground with satisfied accuracy.

Index Terms—Agrawal coupling model, lightning horizontal fields, lightning-induced voltage, 2-D finite-different time-domain (FDTD) method.

I. INTRODUCTION

The calculation of the lightning-induced voltages on overhead lines has been studied for decades [1]–[6], but many authors have even neglected the soil resistivity effect. Now it is very clear that the finitely conducting ground plays an important role in the lightning-radiated horizontal electric field [7], [8] and in the field-line coupling surge propagation on overhead lines [3]. Assuming the ground to be perfectly conducting, Rusck [1], Chowdhuri et al. [9], Liew et al. [10], Jankov [11], Høidalen et al. [12], and Andreotti et al., [13] presented several analytical expressions for estimating the lightning-induced voltages on overhead lines. Taking into account the effect of the finitely conducting soil, Høidalen et al. [14] proposed an approximate formula; however, their formula is complicated and time consuming. In order to simplify the complicated coupling formulas, Barker et al. [15] revised the simplified Rusck’s expression by adding a correction factor of the resistivity effect. Darveniza [16] further presented an empirical formula for estimating the lightning-induced voltages peak value on the overhead lines based on the consideration of the experimental data and theoretical analysis.

Paulino et al. [17] also present an approximate formula for the evaluation of the peak value of lightning-induced voltages, considering finitely conducting soil and using a step waveform current. In their calculation method, the field-line coupling model is an Agrawal field-line coupling model [18], and the calculation of the vertical electric-field component method is proposed by Rusck [1] and the horizontal electric-field component method is proposed by Barbosa and Paulino [4]. The approximate formula is compared with experimental results from Barker et al. [15] and theoretical analysis results from Borghetti et al. [5]. Paulino et al. [17] also used their formula with the probabilistic approach of the IEEE Standard 1410 Guide [19] for assessing the lightning performance of overhead distribution lines considering finitely conducting soil. However, the approximate formula proposed by Paulino et al. [17] is only based on a step waveform current in order to further consider the influence of the current waveform front time on the field-line coupling model. Paulino et al. [20] also present an improved approximate formula, considering the effect of the stroke current front time. They used a trapezoidal lightning return stroke current waveform with a typical 3.8-µs front time and 120-m/µs return stroke velocity. These representative discharges parameter values (e.g., 3.8-µs front time and 120 m/µs return stroke velocity) were validated by Borghetti et al. [5] using randomly variable front-time values and experimental results from Eriksson et al. [21]. Recently, Andreotti et al. [22] developed an analytical approach for lightning-induced voltage calculation and compared the results of their method with results predicted by the analytical expressions proposed by Barker et al. [15]; Darveniza [16]; Paulino et al. [17], [20]; and Høidalen [12].

From before, the approximate formula proposed by Paulino et al. [20] is very important and convenient for estimating the lightning-induced voltages peaks on an overhead line, however, it is only valid under the homogeneously conducting ground. In fact, the real soil is better represented by horizontally or vertically stratified models. Therefore, in this paper, we will revise and extend the approximate formula proposed by Paulino...
Fig. 1. Trapezoidal lightning return stroke current waveform with typical 3.8-μs front time.

et al. [20] to the case of the horizontally stratified ground, and then examine its accuracy by using the 2-D FDTD method and Agrawal coupling model. The transmission-line model (TL) is employed, the return stroke current is a trapezoidal waveform with the front time of 3.8 μs, and the return stroke velocity is 120 m/μs, proposed by CIGRÉ [23] and IEEE Standard 1410 Guide [19], which are the same as Paulino et al. [20].

II. APPROXIMATE FORMULA FOR ESTIMATING THE PEAK VALUE OF LIGHTNING-INDUCED OVERVOLTAGE CONSIDERING THE HORIZONTALLY CONDUCTING GROUND

According to Paulino et al. [20], for a trapezoidal lightning return stroke current waveform with typical 3.8-μs front time (see Fig. 1) and 120-m/μs velocity proposed by IEEE Standard 1410 [19], the induced voltage peak value at the closest point along the line with respect to the stroke point can be expressed by

\begin{align}
V_p &= k_c (V_H + V_S) \\
V_H &= 8.5 I_0 \frac{h}{y^{3/4}} \\
V_S &= -\sqrt{3} (\nu_e)^{1/3} I_0 \sqrt{\frac{\rho}{y}} 
\end{align}

where \( V_p \) is the induced voltage component for the perfect conducting soil, \( V_S \) is the induced voltage component due to the finitely conducting soil, \( I_0 \) is the peak value of the stroke current, \( \rho \) is the homogeneously soil resistivity, \( h \) is the line height, \( y \) is the closest distance between the stroke and the line (\( y < L/10 \) and \( L \) is the length of the overhead line), \( \nu_e \) is the relative velocity of the return stroke \( (\nu_e = \nu/v) \), \( \nu \) is the velocity of the return stroke current propagating along the lightning channel, and \( c \) is the velocity of the light. The factor \( k_c \) is necessary to account for the delay between the voltage peaks given by (2) and (3), and the best value is estimated to be \( k_c = 0.9 \).

The \( V_H \) component in (2) is valid for the case of an infinite line of height \( h \) over the perfectly conducting ground, and the \( V_S \) component in (3) is the induced voltage peak value closest to the lightning discharge if the line is at the finitely conducting ground surface \( (h = 0) \). For the case of horizontally stratified ground, the \( V_H \) component is still valid and is not modified here. In the following section, we will revise and extend the \( V_S \) component to the case of the horizontally stratified ground.

Assuming that the \( V_S \) component depends on the earth conductivity, but is independent of the line height \( h \), therefore, the \( V_S \) component is proportional to the horizontal electric field at the layered ground surface produced by the lightning return stroke. According to Barbosa et al. [24], the total horizontal electric field \( E_L(0,t) \) at the surface of layered earth can be obtained from the horizontal electric field \( E_H(0,t) \) that would exist at the surface of the homogeneously conducting earth with the electric parameters of the first layer by using a recursive [24].

\[ E_L(0,t) = E_H(0,t) + 2 \sum_{i=1}^{\infty} k^i E_H(2h_i,i,t) u(t-2i\tau_0) \]

where the second term in the right side of (4) is the attribution component due to the reflected wave in the boundary between the two layers. As shown in Fig. 2, \( h_1 \) is the depth of the first layer, and the first layer has conductivity \( \sigma_1 \), relative permittivity \( \epsilon_{\sigma_1} \), and permeability \( \mu_0 \), while the second layer has conductivity \( \sigma_2 \), relative permittivity \( \epsilon_{\sigma_2} \), permeability \( \mu_0 \), and infinite depth. If the conductivity of the ground is homogeneous, the underground field is not reflected and the second term in (4) vanishes, so it is also valid for the homogeneously conducting earth. \( \tau_0 \) is the transit time from the surface to the boundary between the layers, that is, \( \tau_0 = h_1/v.u.a(t-2i\tau_0) \) is the Heaviside’s function which is 1 for the positive argument and 0 otherwise. The letter \( i \) is the iteration time.

When the wave reaches the boundary between the two layers, a reflection takes place. \( k \) is the corresponding reflection coefficient. If the displacement currents in the earth are neglected during the reflection, the reflection coefficient proposed by Barbosa et al. [24] is given as follows:

\[ k = \frac{\sqrt{\sigma_1} - \sqrt{\sigma_2}}{\sqrt{\sigma_1} + \sqrt{\sigma_2}} \]
The reflection coefficient given by (5) is a constant, and the reflected wave can be obtained by multiplying the arriving wave by the factor \( k \). Without considering the difference of the attenuation caused by the propagation direction, such as upward or downward between the two layers, we find that the horizontal field peak value \( E_{HF} \) approximately sharply decreases with the depth distance dependence of the exponential function as shown

\[
E_{HF}(2h_1i) - E_{HF}(0) \exp(-2h_1\alpha i) \tag{6}
\]

where \( \alpha \) is the fitted attenuation factor in the exponential function for the propagating field in the first layer. Table I shows the detailed information of the fitted attenuation factor \( \alpha \). In order to obtain the factor \( \alpha \) under different soil conductivities, we employ a 2-D FDTD technique for calculating the field, and the attenuation factor \( \alpha \) is the fitted value. The working space is 1200 m x 2000 m, which is divided into square cells of 1 x 1 m, the time increment is set to 1.66 ns, and the first-order Mur absorbing boundary condition is employed in order to simulate the unbounded space. The simulation domain of the 2-D FDTD technique is shown in Fig. 2. From fitting attenuation factor \( \alpha \) under different soil conductivities, such as 0.001, 0.002, 0.004, 0.006, 0.008, 0.01, 0.02, 0.04, 0.06, 0.08, and 0.1 S/m, we find that the value of \( \alpha \) complies well with (7). As shown in Fig. 3, it is noted that there is good agreement between (7) and our fitted attenuation factor \( \alpha \)

\[
\alpha = 0.3\sqrt{10\sigma} \tag{7}
\]

In Fig. 3, the factor \( \alpha \) increases with the increase of the first-layer conductivity. For example, when the first-layer conductivity is 0.1, 0.01, and 0.001 S/m, the average attenuation factor \( \alpha \) is 0.3 m\(^{-1}\), 0.097 m\(^{-1}\), and 0.03 m\(^{-1}\) within the distances of 400 m from the lightning channel, respectively.

From (4) and (6), the peak value of the horizontal electric field at the surface of the two layers is approximately written as follows:

\[
E_{LF}(0) = E_{HF}(0) + 2\sum_{i=1}^{\infty} k^i E_{HF}(0) \exp(-2h_1\alpha i)
\]

\[
= E_{HF}(0) \left[ 1 + 2\sum_{i=1}^{\infty} k^i \exp(-2h_1\alpha i) \right]. \tag{8}
\]

For the case of the horizontally conducting ground, the \( V_S \) component at the ground surface (\( h = 0 \)) can be revised as follows:

\[
V_y = \sqrt{3} \left( \frac{I_0}{\sigma_1} \right)^{1/3} \times \frac{\rho_1}{y} \times \left[ 1 + 2\sum_{i=1}^{\infty} k^i \exp(-2h_1\alpha i) \right]. \tag{9}
\]

Therefore, from (1), (3), and (9), we can estimate the lightning-induced voltages’ wave peak value \( V_P \) at the closest point of the overhead line, considering the horizontally stratified conducting ground as shown in Fig. 2.

### III. Validity of the Revised Approximate Formulas by Using the 2-D FDTD Method and the Agrawal Coupling Model

In the following sections, in order to validate (1), (2), and (9), and to establish their validity limits, we will test the accuracy of the revised approximate formulas for the case of the horizontally stratified conducting ground by using the 2-D FDTD method and the Agrawal coupling model. However, we first validate our 2-D FDTD method and the Agrawal coupling model by using the results proposed by Paulino et al. [20] in the homogeneously conducting ground, because the latter approach has been validated by experiment data. Paulino et al. [20] proposed a time-domain analysis method for the lightning-induced surges on an overhead line, and their simulated results are compared with the current induced on a nearby 2600-m-long line produced by rocket-triggered lightning and show good agreement, considering the homogeneously conducting ground and a trapezoidal current waveform.
A. Comparison with the Results Proposed by Other Authors

Assuming the same parameters as those in Paulino et al. [20], Fig. 4 shows the comparison between the result predicted by our 2-D FDTD and the Agrawal coupling model and that presented in [20]. Note that our results agree fairly well with those computed by Paulino et al. [20], which show that our 2-D FDTD and the Agrawal coupling model has satisfied accuracy for the trapezoidal current waveform with the front time of 1.0 µs and the return stroke velocity is 120 µs.

B. Validity of our Approximate Formulas

Fig. 5 shows the lightning-induced overvoltage waveform on an overhead line at the point closest to the lightning discharge for the horizontally stratified ground by using our 2-D FDTD method and the Agrawal coupling model, considering a trapezoidal current waveform. The adopted values for the electric parameters of the horizontally stratified conducting ground are given in Table II.

When the upper ground layer has lower conductivity than the lower layer (referred to as Case 1 in Table II), from Fig. 5(a) and (b), it is noted that the induced wave peak increases with the increase of the depth of the upper layer, because the increase of the upper-layer depth causes the total effective surface impedance to increase. However, when the upper ground layer has higher conductivity than the lower layer (referred to as Case 2 in Table II), from Fig. 5(c) and (d), the lightning-induced wave peak decreases with the increase of the depth of the upper layer, because the increase of the depth of the upper layer causes the total effective surface impedance to decrease. With the increase of the depth of the upper layer with higher conductivity, the field attenuates more in the first layer and no or less fields are reflected from the boundary between the layers, and the contribution of the second layer becomes less. Above all, the horizontally stratified ground has a lot of influence on the lightning-induced overvoltage on the overhead line especially for the upper layer with lower conductivity.
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TABLE II
PARAMETERS FOR THE HORIZONTALLY STRATIFIED GROUND

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>$\varepsilon_1$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

TABLE III
VALIDATION OF OUR EXTENDED APPROXIMATE FORMULA BY USING THE 2-D FDTD METHOD AND AGRAWAL COUPLING MODEL FOR CASE 1 IN TABLE II

<table>
<thead>
<tr>
<th>Distance(m)</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1=2m$</td>
<td>$V_p$ (kV)</td>
<td>59.65</td>
<td>37.61</td>
<td>23.87</td>
<td>18.36</td>
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<tr>
<td></td>
<td>$F-A$ (kV)</td>
<td>62.04</td>
<td>40.55</td>
<td>24.03</td>
<td>17.86</td>
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<tr>
<td></td>
<td>Difference</td>
<td>3.85%</td>
<td>7.25%</td>
<td>6.65%</td>
<td>2.8%</td>
</tr>
<tr>
<td>$h_1=5m$</td>
<td>$V_p$ (kV)</td>
<td>63.51</td>
<td>40.33</td>
<td>25.80</td>
<td>19.93</td>
</tr>
<tr>
<td></td>
<td>$F-A$ (kV)</td>
<td>64.71</td>
<td>42.56</td>
<td>25.30</td>
<td>18.90</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>1.85%</td>
<td>5.24%</td>
<td>-1.96%</td>
<td>-5.45%</td>
</tr>
<tr>
<td>$h_1=10m$</td>
<td>$V_p$ (kV)</td>
<td>69.27</td>
<td>44.40</td>
<td>26.88</td>
<td>22.28</td>
</tr>
<tr>
<td></td>
<td>$F-A$ (kV)</td>
<td>68.95</td>
<td>45.67</td>
<td>27.31</td>
<td>20.39</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>-0.46%</td>
<td>2.79%</td>
<td>-5.02%</td>
<td>-9.27%</td>
</tr>
<tr>
<td>$h_1=20m$</td>
<td>$V_p$ (kV)</td>
<td>78.16</td>
<td>50.69</td>
<td>33.12</td>
<td>25.29</td>
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<tr>
<td></td>
<td>$F-A$ (kV)</td>
<td>77.45</td>
<td>51.56</td>
<td>30.81</td>
<td>23.01</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>-0.1%</td>
<td>1.69%</td>
<td>-7.5%</td>
<td>-9.91%</td>
</tr>
</tbody>
</table>

Note: F-A means 2-D FDTD method and Agrawal coupling model, and $h_1$ represent the depth of the upper layer.

Based on the simulation-induced peak value in Fig. 5, we will validate the accuracy of the results predicted by (1), (2), and (9) as follows:

$$\text{Difference}(\%) = \left( \frac{V_{\text{peak - FA}} - V_{\text{peak - formula}}} {V_{\text{peak - FA}}} \right) \times 100\% \quad (10)$$

where $V_{\text{peak - FA}}$ means the peak value of the induced voltage simulated by using 2-D FDTD and the Agrawal coupling model, and $V_{\text{peak - formula}}$ is the peak value predicted by using (1), (3), and (9), and $k_C$ is fitted to be 0.9 for Case 1 and Case 2 in Table II. The iteration number $i$ is equal to 20, because, in most practical cases, as shown in (8), the traveling wave nearly vanishes away after a few reflections if the iteration time is larger than 20. For instance, the wave peak is only about 0.036% of the resulting wave peak value for Case 1 in Table II ($h_1 = 5$ m and $y = 200$ m), and only about 0.0008% for Case 2 ($h_1 = 2$ m and $y = 200$ m). The detailed validation of (1), (2), and (9) is shown in Tables III and IV.

When the upper ground layer has a lower conductivity than the lower layer (referred to Case 1 in Table II), it is found that the difference between two methods is dominantly less than 10% for distances ranging from 50 to 400 m from the lightning channel, and our approximate formula is suitable for approximately estimating the lightning-induced voltage peak value with satisfied accuracy, considering the horizontally stratified ground. However, for the cases of the $d = 400$ m and $h_1 > 5$ m, the difference is larger than 10% and our approximate approach has a lot of error.

When the upper ground layer has higher conductivity than the lower layer (referred to Case 2 in Table II), from Table IV, we can see that for distances ranging from 50 to 400 m from the lightning channel, the difference between the two methods is less than 10% and shows good agreement.

In addition, we also validate the accuracy of our formulas assuming that the soil configuration is 0.1 and 0.001 S/m. For the upper layer with the higher conductivity, the maximum difference between the two methods is still less than 10%. For the upper layer with the lower conductivity, the difference is dominantly within a 10% range except for the cases of $d = 400$ m and $h_1 > 10$ m.

IV. CONCLUSION

In this paper, we have revised the approximate formulas proposed by Paulino et al. [20] for estimating the lightning-induced overvoltage peak value at the overhead line center point closest to the striking point and extended it to the horizontally stratified ground, considering a trapezoidal lightning return stroke current waveform with typical parameters 3.8-$\mu$s front time and 120-m/$\mu$s velocity according to CIGRÉ [23] and IEEE Standard 1410 Guide [19]. We have tested our extended approximate formulas by using the 2-D FDTD method and the Agrawal coupling model, and it is found that the proposed approximate formula in this paper is approximately suitable for estimating the lightning-induced voltage peak with a satisfied accuracy within distances from 50 to 400 m from the lightning channel except for a few cases.

REFERENCES


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