Influence of channel tortuosity on the lightning return stroke electromagnetic field in the time domain

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Abstract

The effect of lightning channel tortuosity on the associated electromagnetic (EM) fields in the time domain is investigated quantitatively from a new point of view. Due to the vanishing of geometric symmetry when taking into account the tortuosity of lightning channels, it is necessary to calculate the associated EM fields in the Cartesian coordinate system. An efficient technique is adopted to produce tortuous lightning channels and then the fractal dimension of the channel is calculated automatically by a program. The results show that at close distances, the dependence of EM fields on azimuth is significant, the ratio of MF peaks at two coaxial points with the channel base may reach about 4 and that of the EF may be larger than 2. With increased distance, the dependence of waveform on azimuth weakens. At distances of 10 km and 100 km, the current peaks derived from the EF peaks show good consistency with the current peak adopted, with an uncertainty less than 25%.

Keywords:
Lightning channel tortuosity
EM field
Return strokes
Retardation effect

1. Introduction

Lightning electromagnetic field pulses (LEMP) can be coupled with electrical circuits and electronic systems and produce transient overvoltage, which may cause destruction of electronic devices, and outages of power supply and telecommunication systems. Therefore the study of LEMP from different lightning processes, especially return strokes, has important practical applications in the protection of lightning-induced transient EM pulses. To know the lightning return stroke fields at different distances, several return stroke models have been established (Rakov and Uman, Chap. 12, 2003). These models commonly assume that the lightning current is contained in a straight and vertical channel with a negligible cross-section, and the tortuosity of lightning is not taken into account in these models. But obviously, the real lightning channel is usually characterize by tortuosity (Qie and Kong, 2007; Kong et al., 2008). Hill (1968), using high speed photography, analyzed statistically the tortuosity of the return stroke channel and found that the main channel is characterized by tortuosity on a scale from 1 m to over 1 km with the channel direction azimuth normally distributed with a mean of 16°.

Fig. 1. Schematic of a tortuous lightning channel and the length of channel, L'(t), contributing to the field at point P at time t.
Table 1
Values of the parameters of current waveform at the lightning channel base adopted in the following simulation

<table>
<thead>
<tr>
<th>Cn (kA)</th>
<th>η</th>
<th>t1 (μs)</th>
<th>t2 (μs)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.73</td>
<td>0.15</td>
<td>3.0</td>
<td>2</td>
</tr>
</tbody>
</table>

Previous work concerning the effect of tortuosity on EM fields mainly focused on the radiation EM waveform in the time domain and its components in the frequency domain (LeVine and Meneghini, 1978), and the relationship between the fractal dimension of the discharge channel and the fractal dimension of the generated EM fields in the time domain (Vecchi et al., 1994; Lupo et al., 2000). Due to the disappearance of the geometric symmetry produced by the tortuosity of the lightning channel, how do the EM fields vary with the azimuths of the observation sites?

In this paper, explicit expressions of EM fields related to the tortuous channel are given in the Cartesian coordinate system. Using the numerically simulated fractal lightning channels, we show the dependence of return stroke EM fields on azimuths and the influence played on such dependence by distance between the EM source and observation point.

2. Calculation of the EM fields produced by a tortuous channel

2.1. The expressions of EM fields and the adopted lightning return stroke model

Uman et al. (1975) and Master and Uman (1983) outlined a technique to determine the electric and magnetic fields in the time domain produced by any arbitrary time-varying current propagating along a straight antenna in spherical and cylindrical coordinate systems, respectively. From then on, many papers modeled the lightning return stroke channel as a straight antenna to calculate the fields (e.g., Thottappillil et al., 1998). When the tortuosity of the channel is taken into account, the spherical or cylindrical symmetry vanishes, and then the whole lightning channel can be regarded as a fractal

Fig. 2. Return stroke current pulse at the lightning channel base.

Fig. 3. Lightning channel projections on the x-z and y-z plane and its 3-dimensional representation. The dimensions of the two projections are, respectively, 1.20 and 1.15.
antenna composed of many single line radiators in the Cartesian coordinate system. As shown in Fig. 1, the vector potential at \( P(x, y, 0, t) \) from the time-varying current in \( d\vec{l} \) at source \( S(x', y', z', t') \) can be found as

\[
\vec{A}(P, t) = \frac{\mu i(s, t-R/c)}{4\pi} d\vec{l}
\]  

(1)

With the Lorentz condition, the scalar potential can be written as

\[
\phi(P, t) = -c^2 \int_0^t \nabla \cdot \vec{A} \, dt
\]  

(2)

where \( t_0 \) is the time at which a sensor first senses the source. Then

\[
\mu \vec{H} = \nabla \times \vec{A}
\]  

(3)

\[
\vec{E} = -\nabla \Phi - \frac{\partial}{\partial t} \vec{A}
\]  

(4)

Eqs. (3) and (4) can be written in Cartesian coordinates as

\[
\vec{H}(x, y, 0, t) = \frac{1}{4\pi} \left\{ \frac{i(s, t-R/c)}{R^3} + \frac{1}{cR^2} \frac{\partial}{\partial t} \frac{i(s, t-R/c)}{R} \right\} \, d\vec{l} \times \vec{R}
\]  

(5)

\[
\vec{E}(x, y, 0, t) = \left\{ \frac{\vec{R}}{R} \frac{i(s, t-R/c)}{R^3} \, dt + \frac{3}{cR^2} \frac{i(s, t-R/c)}{R} \, dt + \frac{1}{cR} \frac{\partial}{\partial t} \frac{i(s, t-R/c)}{R} \, dt \right\}
\]  

(6)

where \( \vec{R} = (x-x')\hat{a}_x + (y-y')\hat{a}_y + (-z)\hat{a}_z \), \( d\vec{l} = l\hat{a}_x + m\hat{a}_y + n\hat{a}_z \), \( c \) is the speed of light, \( \mu \) and \( \epsilon \) are the permeability and permittivity of air, respectively. Details of this ratiocination are similar to those of Uman et al. (1975) and Master and Uman (1983).

For a vertical and straight lightning channel with one end fixed at ground, the length of channel contributing to the field at point \( P \) at time \( t \), \( L'(t) \), is given in the analytical expression (25) of Thottappillil et al. (1998). But in the case of a tortuous channel as shown in Fig. 1, the contributing length \( L'(t) \) needs to be calculated by the numerical solution of Eq. (7) for every step.

\[
L'(t) = v \cdot t - v \cdot R(L(t))/c
\]  

(7)

where the upward return stroke front speed \( v \) is assumed to be constant at \( 1.5 \times 10^6 \) m/s (Rakov and Uman, chap. 4, 2003).

The total EM field at \( P(x, y, 0, t) \) at time \( t \) are the vectorial summation of the fields produced by single line radiators composing apparent length \( L'(t) \)

\[
\vec{H}(x, y, 0, t) = \sum_{i=1}^{n} \vec{H}
\]  

(8)

\[
\vec{E}(x, y, 0, t) = \sum_{i=1}^{n} \vec{E}
\]

The modified transmission line model with linear decay (MTLL) (e.g., Thottappillil et al., 2007) is adopted as the return stroke model in the following EM field simulation. Taking into account the tortuosity of the lightning channel, the MTLL model may be written as

\[
I(t', t) = I_0(t-t'/v) \times (1-I/L_{tot})
\]  

(9)

where \( I(S) \) is the tortuous channel length from the source point \( S \) to the base of the return stroke channel, as shown in Fig. 1 and \( L_{tot} \) is the total length of the tortuous return stroke channel. The return stroke current waveform at the channel base \( I_0(t) \) is approximated by the Heidler function (Heidler, 1985)

\[
I_0(t) = \frac{C_n ((t/t_1)^n - 1)}{\eta (t/t_1)^{n/2} + 1} \exp(-t/t_{tr})
\]  

(10)

The values of the parameters shown in Table 1 have been used in the simulations presented in Section 3. Based on these values in Table 1, Fig. 2 shows the current waveform at the channel base corresponding to Eq. (10).

According to statistical results on the frequency spectra of lightning return strokes (Willett et al., 1990), the time step

\[ a \]

\[ b \]

Fig. 4. Waveforms of z-component of EF (a) and amplitude \( \sqrt{H_x^2 + H_y^2} \) of MF (b) evaluated at the ground at a distance of 100 m and different azimuths with respect to the base of the channel. The zero point of the \( T \)-axis is the start time of the return stroke current in Fig. 2.
chosen is 0.1 μs, so the frequency range of the signals produced in this paper is from dc to 5 MHz.

2.2. Calculation of tortuous channels’ fractal dimension and production of tortuous lightning channels

Unlike the box-counting algorithm to calculate the channel fractal dimension (Lupo et al., 2000; Kawasaki and Matsuura, 2000), Sevcik (1998) derived a method for calculating the approximate fractal dimension \( D \) from a set of \( N \) values of points \( y_i \) sampled from a waveform between time zero and \( t_{\text{max}} \). The waveform was subjected to a double linear transformation, and thus became normalized. The fractal dimension of the waveform is then approximated by \( D \) as

\[
D = 1 + \frac{\ln(L) + \ln(2)}{\ln(2N')} (2N')
\]

where \( L \) is the length of the curve in the unit square and \( N' = N - 1 \). The approximation becomes better as \( N \) increases. Compared with the box-counting algorithm, this method can be operated by a computer program without manual counting.

A program originally used as a direct convolution algorithm to create a fractal Brownian signal is utilized in this paper. The fractal Brownian signal can share similar fractal geometry characteristics with a lighting main channel when the parameters of the program are properly chosen, although no physical interpretation can be given so far. The fractal dimension of a fractal Brownian signal is calculated by Sevcik’s method. The fractal dimension can be set to 1.1–1.4, which is the fractal dimension range of the lightning channel (Kawasaki and Matsuura, 2000). Two series of fractal Brownian signal, whose fractal dimension is 1.20 and 1.15, respectively, are used to substitute for the projections of a lightning main channel on the \( x-z \) and \( y-z \) plane (refer to Fig. 3).

3. EM Fields produced by a simulated tortuous lightning channel

Fig. 3 shows the two projections of simulated tortuous channels and their three-dimensional representation. The EM fields generated by the tortuous channel in Fig. 3 traveled by the current waveform (9) and (10) are examined. Figs. 4–7

Fig. 5. Same as Fig. 4 but at a distance of 1 km.

Fig. 6. Same as Fig. 4 but at a distance of 10 km.
show the $z$-component $E_z$ of the electric field (EF) and the amplitude $\sqrt{H_x^2 + H_y^2}$ of the magnetic field (MF), calculated at four different distances, 100 m, 1000 m, 10 km, and 100 km, from the base of the channel, respectively, for the case of point $P(x,y,0,t)$ on the perfectly conducting ground. At every distance, $E_z$ and $\sqrt{H_x^2 + H_y^2}$ are calculated at 8 different azimuths with respect to the channel base.

In Fig. 4a, when azimuth differences are not larger than $3\pi/4$, the EF waveforms among these azimuths somewhat resemble with each other. The similarities of EF waveforms disappear when azimuth differences are bigger than $3\pi/4$, e.g., the waveform at $\varphi = \pi/4$ is obviously not similar to that at $\varphi = \pi$, the peak EF at $\varphi = \pi$ is about 2.1 times of that at $\varphi = \pi/4$, etc. In Fig. 4b, the MF waveforms show an even stronger dependence on azimuth, e.g., the peak value of MF at $\varphi = 3\pi/2$ is about 3.9 times that of at $\varphi = 2\pi$. On the other hand, in Fig. 4, at some special azimuths, $\pi$ and $5\pi/4$, the EF waveforms resemble that of the return-stroke current at the channel base adopted (Fig. 2), and the rise time of the EF approaches that of the return stroke current. But it should be noted that this characteristic may not be found from the EF waveforms at other azimuths. The relationship between the MF waveforms and the return stroke current pulse adopted is similar to the case of the EF.

As shown in Figs. 5–7, the dependence of EF or MF waveforms on azimuth weakens with increase of distance. In Figs. 6 or 7, the ratio of peak EF values at any two azimuths is between 0.8 and 1.3. Using the TLM (e.g. Lin et al., 1980), the current peaks derived from the EF peaks at 10 km and 100 km have good consistency with the current peak adopted, with an uncertainty <25%. But the dependence of MF on azimuth is still prominent; the ratio of MF peaks at any two azimuths is within 0.3–2.9. On the other hand, each of the EF waveforms contains a series of pulses with durations of about 30 μs, and the similarity between the EF field waveforms and the return stroke current pulse decreases. As remarked by Lupo et al. (2000), the rise times of the EF and of the current pulse seem to be uncorrelated. As a consequence it is impossible to derive important lightning parameters, such as current pulse shape, rise time and velocity of propagation, from fields generated at great distances by tortuous channels.

Some explanations for the above descriptions can be derived from the EM field expressions (5), (6) and (8). At close distances, as the observation’s azimuth changes, the angle between the vector $\vec{R}$ and $\vec{d}t$ may change significantly. So from Eqs. (5), (6), and (8), the EM fields may cancel at some points and enhance at other points. This gives birth to the strong dependence of field waveform on azimuth. At great distances, the angle between the vectors $\vec{R}$ and $\vec{d}t$ changes slightly as the observation’s azimuth changes, so the dependence of field waveform on azimuth is eliminated.

4. Summary

Explicit expressions of general EM fields related to a tortuous channel are given in the Cartesian coordinate system. Based on a lightning channel with fractal dimension 1.15 and 1.2 and a given return stroke current pulse, the related EM field waveforms are produced at 32 different “observation points” on the ground with respect to the lightning channel base. The results of the calculation show that at close distances, the field waveform is dependent strongly on azimuth. At a distance of 100 m, the ratio of the peak EF values at two azimuths may be bigger than 2; in the case of the MF, the ratio may reach about 4. With increase of distance, the dependence of waveform on azimuth weakens. At distances of 10 km and 100 km, the current peaks derived from the EF peaks show good consistency with the current peak adopted, with an uncertainty <25%. On the other hand, the correlation between the shapes of the field waveform and of the return stroke current pulse adopted is weaker.

Acknowledgments

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